1 Przyspieszenie

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{\vec{F}}{m} \quad \left[\frac{m}{s^2}\right] \tag{1}$$

$$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} \tag{2}$$

$\vec{a}$ – przyspieszenie, a – przyspieszenie (wielkość skalarna), $\vec{r}$ – położenie punktu materialnego, $\vec{v}$ – prędkość punktu materialnego (pochodna położenia po czasie), $\frac{d\vec{v}}{dt}$ – pochodna prędkości po czasie, $\frac{d^2\vec{r}}{dt^2}$ – druga pochodna położenia po czasie, $\vec{F}$ – wypadkowa siły działającej na ciało, m – masa ciała, $\frac{dv}{dt}$ – przyspieszenie w ruchu prostoliniowym, $\frac{\Delta v}{\Delta t}$ – przyspieszenie w ruchu jednostajnie zmiennym

2 Air density

$$r = \frac{pM}{RT} \tag{3}$$

M – the molecular weight of air: 0.028964 kg/mol, R – universal gas constant: 8.31447 J/(mol/K), T – altitude temperature in Degrees C

$$T = T_0 + Lh \tag{4}$$

T_0 – the standard temperature at sea level: 288.15 K (15 C), L – adiabatic lapse rate for dry air: -0.0065 K/m, h – the altitude above sea level in meters

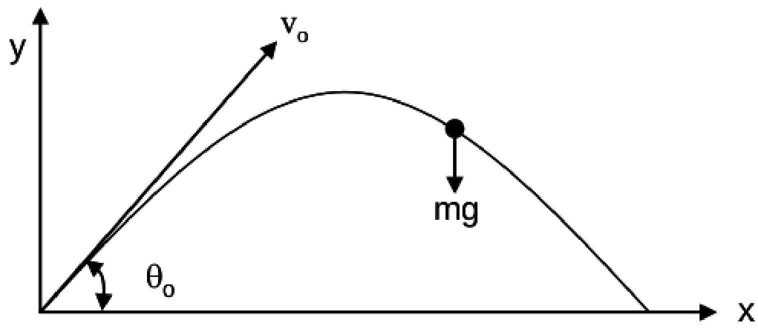
$$p = p_0 \left(1 + \frac{Lh}{T_0}\right)^{\frac{gM}{R\cdot(-L)}} \tag{5}$$

g – the average gravitational acceleration at the Earth's surface: 9.80655 m/s^2, p_0 – standard pressure at sea level: 101,325 kg/ms^2

3 Equation of motion

$$v = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} \tag{6}$$

4 Zero drag model

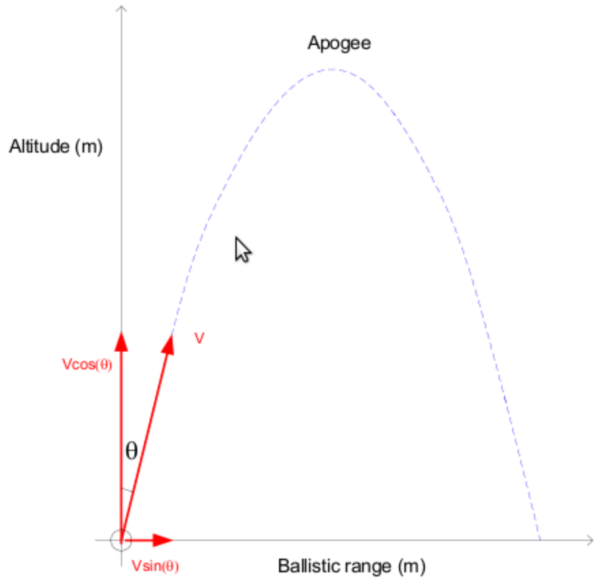


$$t = 0, x = 0, y = 0, v_{ox} = v_0 \cos\theta_0, v_{oy} = v_0 \sin\theta_0 \tag{7}$$

t – time, x – horizontal distance, y – height, v_0 – muzzle velocity, θ_0 – quadrant elevation

$$x = v_0 \cos\theta_0 t \tag{8}$$

$$y = v_0 \sin\theta_0 t - \frac{gt^2}{2} \tag{9}$$

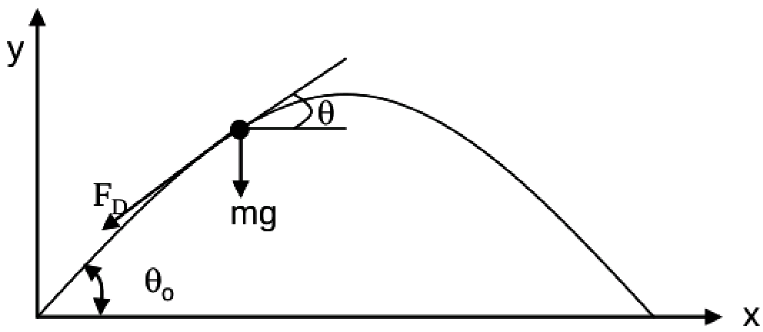


$$V_{vertical} = V \cos\theta \tag{10}$$

$$V_{horizontal} = V \sin\theta \tag{11}$$

5 Point mass model

5.1 Two Degree of Freedom Point Mass Model



$$a_x = \frac{-F_D \cos\theta}{m} \tag{12}$$

$$a_y = -g - \frac{F_D \sin\theta}{m} \tag{13}$$

a – acceleration of the projectile, θ – angle between the velocity vector and horizontal plane, g – gravitational constant, m – mass of projectile

$$a = \frac{dv}{dt} = \frac{v(t + dt) - v(t)}{dt} \tag{14}$$

v – velocity of the projectile

$$v_x(t + dt) = a_x dt + v_x \tag{15}$$

$$v_y(t + dt) = a_y dt + v_y \tag{16}$$

$$X(t + dt) = v_x dt + X \tag{17}$$

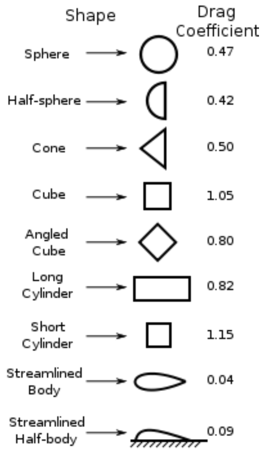
$$Y(t+dt) = v_y dt + Y \tag{18}$$

X – displacement in the range direction, Y – displacement in the vertical direction

5.2 Three Degree of Freedom Point Mass Model

Contains drag force and wind effect. Magnus effect and the Coriolis effect are ignored.

5.2.1 Drag force



Measured Drag Coefficients

$$C_d = \frac{2F_d}{\rho v^2 A} \tag{19}$$

C_d – drag coefficient, F_d – drag force (the force component in the direction of the flow velocity), ρ – the mass density of the fluid, v – the speed of the object relative to the fluid, $A = \pi r^2$ – the reference area

$$F_D = \frac{1}{2} C_D S \rho V^2 \quad [N] \tag{20}$$

$$F_D = \frac{1}{2} C_D \frac{A}{M} \rho V^2 \quad [N] \tag{21}$$

$$F_D = -\frac{1}{8} C_D \rho \pi d^2 V^2 \tag{22}$$

$S = \frac{\pi d}{4}$, C_D – drag coefficient, d – projectile caliber (projectile reference diameter), $\rho = 1.25\frac{kg}{m^3}$, V – projectile velocity with respect to the atmosphere, $\frac{A}{M}$ – area to mass ratio, $\frac{1}{2}\rho V^2$ – dynamic pressure

$$F_{D,R} = F_D \cos\varphi \tag{23}$$

$$F_{D,D} = F_D \sin\varphi \tag{24}$$

$F_{D,R}$ – drag force in the range direction, $F_{D,D}$ – drag force in the deflection direction, φ – angle between the projectile axis and trajectory

The horizontal and vertical components of the acceleration on the projectile in range:

$$a_x = \frac{-F_{D,R}\cos\theta}{m} \tag{25}$$

$$a_y = -g - \frac{-F_{D,R}\sin\theta}{m} \tag{26}$$

Acceleration of the projectile in the deflection direction:

$$a_{D,w} = \frac{F_{D,D}}{m} \tag{27}$$

5.2.2 Wind effect

$$D_H = W\left(T - \frac{R}{V_0}\right) \tag{28}$$

D_H – horizontal deflection due to the crosswind, W – crosswind velocity, T – time of flight to range R, R – range to target, V_0 – muzzle velocity

$$\vec{v}_r = \vec{v} + \vec{w} \tag{29}$$

$\vec{v}_r$ – relative velocity of the projectile, $\vec{w}$ – velocity of the wind
The drift due to the wind effect:

$$Z_w = \frac{1}{2}a_{D,w}t^2 \tag{30}$$

$$F_w = -C_w v_w \tag{31}$$

C_w – the drag coefficient, v_w – the wind speed

Defining the wind direction as measured by angle y following formulas to determine the x and z components of the wind force on the ball is used:

$$F_{xw} = F_w \cos y = -\cos y(C_w v_w) \tag{32}$$

$$F_{zw} = F_w \cos y = -\sin y(C_w v_w) \tag{33}$$

5.2.3 Drift due to Rotating Projectile Effects

If a constant acceleration can be assumed the drift due to the projectile effects can be estimated by:

$$Z_p = \frac{1}{2}a_{D,p}t^2 \tag{34}$$

$a_{D,p}$ – the estimated cumulative lateral acceleration of the projectile due to the gyroscopic effect determined from actual data for a particular projectile

5.2.4 Total deflection experienced by a projectile using the 3 DOF model

$$Z = Z_w + Z_p \tag{35}$$

6 Ballistic Coefficient

BC's may be given in different units: $\frac{lb}{in^2}$ or $\frac{kg}{m^2}$

$$BC = \frac{weight/7000}{i \cdot cal^2} \quad \left[\frac{lb}{in^2}\right] \tag{36}$$

weight – bullet weight in grains, i – form factor (G1 or G7) (for G1 i=1), *cal* – bullet caliber in inches

Bryan Litz,“Understanding Long Range Bullets Part I: The Nature of Scale” Precision Shooting, May 2007

Robert L. McCoy, “Modern Exterior Ballistics” Schiffer Publishing, Ltd., Atglen, PA, 1998.

The BC formula gives the ratio of ballistic efficiency compared to the standard G1 model projectile. The standard G1 projectile originates from the “C” standard reference projectile (a 1 pound (454 g), 1 inch (25.4 mm) diameter projectile with a flat base, a length of 3 inches (76.2 mm), and a 2 inch (50.8 mm) radius tangential curve for the point) defined by the German steel, ammunition and armaments manufacturer Krupp in 1881. By definition, the G1 model standard projectile has a BC of 1.

A bullet with a high BC will travel farther than one with a low BC.

$$BC = \frac{R_1 - R_0}{\log_e(V_0/V_1) * 8000} \tag{37}$$

BC – ballistic coefficient, R_0 – Near range [yards], R_1 – Far range [yards], V_0 – velocity at R_0 $[Ft/s^2]$, V_1 – velocity at R_1 $[Ft/s^2]$

$$BC_{ph} = \frac{M}{C_d A} = \frac{\rho l}{C_d} \tag{38}$$

BC_{ph} – ballistic coefficient as used in physics and engineering

$$BC_{bt} = \frac{SD}{i} = \frac{M}{id^2} \tag{39}$$

BC_{bt} – ballistic coefficient used in bullet ballistics

A – cross–sectional area, C_d – drag coefficient, i – form factor (drag coefficient of the bullet/drag coefficient of G1 model bullet; G1 drag coefficient = 0.5190793992194678), M – Mass of object (lb or kg), d – diameter of the object (in or m), l – body length, ρ – average density, $SD = \frac{m}{d^2}$ – sectional density (mass of bullet in pounds or kilograms divided by its caliber squared in inches or meters; units are lb/in^2 or kg/m^2)

$$i = \frac{C_B}{C_G}, \quad (C_G \sim 0.5191) \tag{40}$$

C_B – drag coefficient of the bullet, C_G – drag coefficient of the G1 model bullet

$$BC = \frac{mC_G}{C_B d^2} \tag{41}$$

$$C_d = C_g \frac{m}{BCd^2} \tag{42}$$

Tabulated BC values are defined in terms of the G1 projectile described in pounds mass and inches diameter. To work in SI units redefine the G1 projectile in kg/m2 by adding the conversion factor 0.0014223:

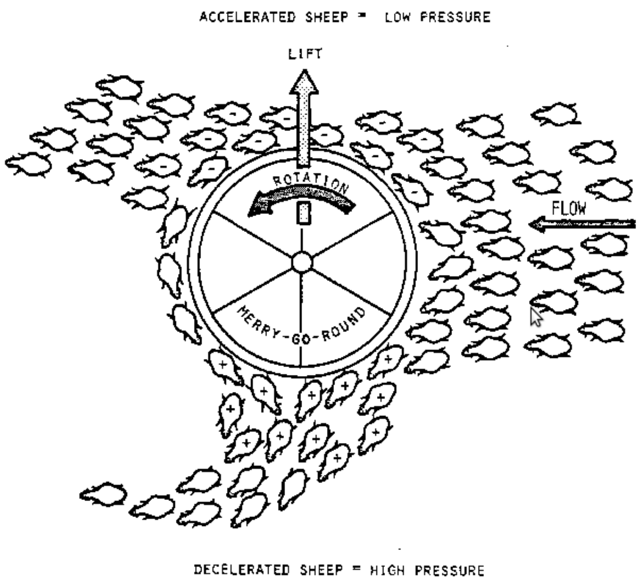
$$C_d = 0.0014223 C_g \frac{m}{BCd^2} \tag{43}$$

where mass is in kilograms and diameter is in meters.

7 Pellet spin

Pellet exiting a 1:12 twist at 900 fps is spining 54,000 rpm. Out at 180-200 yards, it has slowed to about 400 fps but is spinning about 40,000 rpm.
Same pellet shot at 400 fps at the muzzle is spinning 24,000 rpm.

8 Magnus force



$$F_M = C_L r v^2 A \tag{44}$$

r – air density $[kg/m^3]$, v – velocity [m/s], A – the cross-sectional area of the projectile
C_L – Dr. Dyrkacz's polynomial to calculate lift coefficient: C_L = (-0.0020907 -0.208056226 * (V/U) + 0.768791456 * (V/U)^2 - 0.84865215 * (V/U)^3 + 0.75365982 * (V/U)^4) / (1 - 4.82629033 * (V/U) + 9.95459464 * (V/U)^2 - 7.85649742 * (V/U)^3) + 3.273765328 * (V/U)^4) where V is the rotational velocity and U is the linear velocity

$$\vec{F}_L = \frac{1}{2} C_L \rho A v^2 \tag{45}$$

$\vec{F}_L$ – Magnus force, C_L – lift coefficient (C_L depends on the rate of spin, being roughly proportional to the rate of spin), ρ – the density of the air, A – cross-sectional area of the ball

$$C_L = \frac{1}{2 + \frac{v}{v_{spin}}} \tag{46}$$

$v_{spin} = R\omega$ – peripheral speed of the ball, R is its radius and ω is the angular speed about a horizontal axis perpendicular to the path of the ball

$$F_M = 4\pi\rho V \omega R^3 \tag{47}$$

$$\bar{p}_0 = \frac{2\pi V_0}{\lambda D} \quad \left[\frac{rad}{s}\right] \tag{48}$$

$\bar{p}_0$ – initial spin rate, V_0 – the initial firing velocity $[m/s]$, λ – the rifling twist rate at the M2 gun muzzle [calibers per turn], D – the reference diameter of the bullet type [m]

$$F_l = (dvr_4 \ av \ 2di^2)(2r) \tag{49}$$

d – density of air at 60°=2.37·10^{-3}, v – velocity, r – ball radius, av – angular velocity in radians per second (if you have spin in rpm, them av = (2pi n)/60

```
Fm = 2pi(p)(w)(vx)(h r^2)

where p = density at 25 degrees C, w~= angular velocity, vx -
horizontal velocity and h r^2 = height of cylinder multiplied by the
radius squared.

this formula is for a~cylinder.
i've rearranged it:
Fm = 2(p)(w)(vx)(pi r^2 h)

now, as far as i~can remember, pir^2h is the formula for the volume of
a~cylinder...
as im looking at golf, i've taken out that formula and replaced it
with the formula for the volume of a~sphere:

Fm = 2(p)(w)(vx)(4/3pi r^3)

which eventually, returns the pi to the front:

Fm = 8pi(p)(w)(vx)(r^3)
3

^^ thats divided by 3, btw.
```

For a spinning circular cylinder moving through a fluid:

$$F_L = 2\pi\rho L v r^2 \omega \tag{50}$$

v – speed, L – the length of the cylinder, r – its radius, and ω – is its angular velocity in radians per second (rad/s), $\omega = 2\pi n$ where n is in [rps]

For a spinning sphere moving through a fluid:

$$F_L = \frac{2\pi^2 \rho v r^4 \omega}{2r} \tag{51}$$

r – the radius of the sphere

9 Mach number

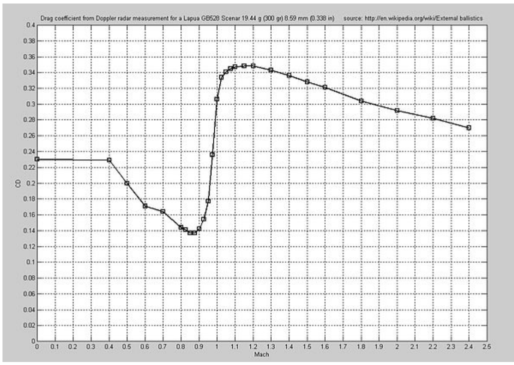
$$M = \frac{V}{a} \tag{52}$$

M – the Mach number (dimensionless), V – the velocity of the source relative to the medium, a – speed of sound in the medium
At Standard Sea Level conditions (corresponding to a temperature of 15 degrees Celsius), the speed of sound is 340.3 m/s (1116 ft/s).

Doppler radar measurement results for a Lapua GB528 Scenar 19.44 g (300 gr) 8.59 mm (0.338 in) calibre very-low-drag bullet:

M	0.000	0.400	0.500	0.600	0.700	0.800	0.825	0.850	0.875	0.900	0.925	0.950	0.975	1.000
C_d	0.230	0.229	0.200	0.171	0.164	0.144	0.141	0.137	0.137	0.142	0.154	0.177	0.236	0.306

M	1.025	1.050	1.075	1.100	1.150	1.200	1.300	1.400	1.500	1.600	1.800	2.000	2.200	2.400
C_d	0.334	0.341	0.345	0.347	0.348	0.348	0.343	0.336	0.328	0.321	0.304	0.292	0.282	0.270



10 Golf ball aerodynamics

P. W. Bearman and J. K. Harvey (P. W. Bearman and J. K. Harvey – Golf ball aerodynamics, Aeronautical Quarterly, pp. 112 – 122, May 1976)

$$\ddot{x}_n = -\frac{\rho \cdot S}{2 \cdot m}\left(\dot{x}_n^{\,2} + \dot{y}_n^{\,2}\right)\left(c_D \cdot \cos(\alpha) + c_L \cdot \sin(\alpha)\right)$$

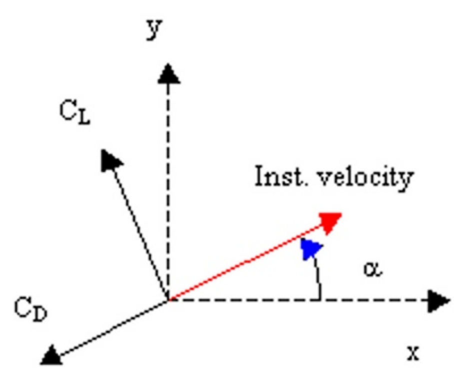
$$x_{n+1} = x_n + \dot{x}_n \cdot dt + \tfrac{1}{2}\ddot{x}_n \cdot dt^2$$

$$\dot{x}_{n+1} = \dot{x}_n + \ddot{x}_n \cdot dt$$

$$\ddot{y}_n = \frac{\rho \cdot S}{2 \cdot m}\left(\dot{x}_n^{\,2} + \dot{y}_n^{\,2}\right)\left(c_L \cdot \cos(\alpha) - c_D \cdot \sin(\alpha)\right) - g$$

$$y_{n+1} = y_n + \dot{y}_n \cdot dt + \tfrac{1}{2}\ddot{y}_n \cdot dt^2$$

$$\dot{y}_{n+1} = \dot{y}_n + \ddot{y}_n \cdot dt$$



p – density of air at sea level=1.225 kg/m^3, S – on stream surface (=πr^2, r=20.55 mm)=1.326710^{-3} m^2, m – is mass of the ball=0.050 kg, a – the instantaneous angle between the instantaneous velocity and the horizon, CD – the drag coefficient which has the same direction as the instantaneous velocity, CL – the lift coefficientm which direction is perpendicular to the instantaneous velocity, dt – the time step ('step size'), x – the coordinate in the direction of the range, y – the coordinate in the direction of the height

11 Gyroscopic effect

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Bryan Litz (Applied Ballistics for Long Range Shooting) gives the
formula for the Gyroscopic (spin) drift as:

Drift = 1.25(Sg+1.2)tof^1.83

Sg = Gyroscope stability factor (for the example I~use the 175 SMK
with a~Sg of 1.5)

tof = time of flight

Spreadsheet format: Drift=1.25(1.5+1.2)*tof^1.83

This gives the drift of the 30 cal 175 grn SMK @2800 FPS a~drift of
7.19 inches at 1000 yards.
```

(53)